Topic Study Group 17

Problem Posing and Solving in Mathematics Education

Tin-Lam Toh[1], Manuel Santos-Trigo[2], Puay Huat Chua[3], Nor Azura Abdullah[4], and Dan Zhang[5]

ABSTRACT This report presents a summary of the content of the various presentation by the participants in ICME-14 under the Topical Study Group 17: Problem Posing and Solving in Mathematics Education. Some trends in the research on problem solving and problem posing are identified through this study. Other areas which were less explored were also highlighted

Keywords: Problem solving; Problem posing; Processes; Teaching and learning; Teacher education.

1. TSG-17 — A Brief Introduction

1.1. TSG-17 on Problem Solving/Problem Posing

The TSG-17 on Problem Solving (PS) and Problem Posing (PP) attracted large numbers of paper submissions. After the review process and confirmed registration of the participants, there were 35 paper presentations out of which 6 were long paper presentations and 29 were classified as short orals. There were a good mix of presenters from all over the world from the five continents. Hence, the views and trends in this study is a balanced views from researchers from all parts of the globe.

1.2. Questions on PS/PP to be addressed by the TSG

The focus of the TSG-17 is on four aspects of PS/PP: (1) the teaching and learning of mathematics in relation to PS/PP; (2) the enactment of PS/PP in the mathematics classroom; (3) teacher education and (4) the use of digital technologies in PS/PP activities. Mathematical PS has been the focus of a long line of inquiry in mathematics education for more than half a century, which gets back to as far as the publication of the seminal work *How to Solve It* of George Pólya (1945). With this attention on PS, the mathematics

[1] National Institute of Education, Nanyang Technological University, Singapore. E-mail: tinlam.toh@nie.edu.sg
[2] Centre for Research and Advanced Studies, Cinvestav-IPN; Mathematics Education department, Mexico City, 07360/ Mexico. E-mail: msantos@cinvestav.mx
[3] National Institute of Education, Nanyang Technological University, Singapore. E-mail: puayhuat.chua@nie.edu.sg
[4] Universiti of Brunei Darussalam, Brunei Darussalam. Email: azura.abdullah@ubd.edu.bn
[5] Beijing Academy of Educational Sciences, Beijing, China. Email: zhangdanbj99@163.com

curricula around the world have placed PS as the heart of the national mathematics curriculum in many countries. The four-phase Pólya's problem solving model became well-known and is an icon for PS. Schoenfeld (1985) introduced his framework to further enhance Pólya's model. Although PS has been in attention for many years, its emphasis has not diminished. For example, in the Singapore context, PS has been the heart of the school mathematics curriculum. Despite the regular curricular revision, the PS framework remains unchanged, with the new initiatives of mathematics education introduced by the Singapore Ministry of Education serving to unpack and elucidate the various aspects of the PS framework. In China, PS and PP are highly emphasized in the national curriculum standard and are the main objectives of the national curriculum. PS/PP are widely used in the mathematics classrooms, although it is recognized that there is still the need to strengthen teachers' ability of encouraging students engage in PS and PP. PS has continued to attract researchers' attention in areas such as the enactment of PS in the mathematics classroom (Toh et al., 2008a, 2008b). For example, in Brunei schools, teachers are studying effective strategies to enact PS in the mathematics classroom. Not only has PS continued to attract attention of researchers even recently, new trends on PS have continued to emerge (Liljedahl and Santos-Trigo, 2019). In particular, in this technological era, studies on how PS can ride on the affordance of technology to enhance students' PS ability emerges (e.g., Santos-Trigo, 2019). Recently in Mexico, curriculum reforms at elementary and high school levels recognize that both PS and PP activities are central for students' development of mathematical thinking. Thus, in the post-pandemic instructional approaches, students are encouraged to always look for different ways to represent, explore, and solve problems and to constantly pose and pursue new questions or problems.

By comparison, PP is a much younger field of inquiry in mathematics education. A quick search of the education literature shows clearly that attention to this topic has grown rapidly in recent decades. Researchers and educators in many countries have incorporated PP as a research and instructional focus respectively. The juxtaposition of these two topics PP and PS in this TSG thus merges a mature field of inquiry with a more nascent one.

2. Content of the Paper Presentation

Although there are 35 paper presentation, we were able to identify three main trends of PS and PP throughout all the paper presentations. In giving a brief description of the papers, the session and the order of the speakers are presented in Tab. 1 (on the next page). There were four sessions: Session 1 (13 July, 14:30 to 16:30 GMT+8), Session 2 (14 July 19:30 to 21:30 GMT+8), Session 3A (17 July, 14:30 to 16:30 GMT+8) and Session 3 (17 July, 21:30 to 23:30 GMT+8). As this is a synchronous online presentation, the presentation slots were arranged mainly based on the time zone of the speakers, instead of the content. In re-classification of the content of the paper, four trends were visible: (1) teaching and learning in relation to PS/PP; (2) processes involved in PS/PPS; (3) teacher education in relation to PS/PP; and (4) textbook analysis and comparative studies on PS/PP. The details are discussed in the following subsections.

Tab. 1. List of papers presented

Paper and author(s)
Session 1
[1] Analysis on creating problem situation in middle school mathematic teaching. ***Peijun Zheng*** (China).
[2] Historical comparison and analysis of problems and problem-sovling in middle school mathmatics textbooks. *Rong Wang and* ***Cuiqiao Wang*** (China).
[3] Problem posing among Pre-service and in-service mathematics teachers. ***Ma Nympha Beltran-Joaquin*** (Philippines).
[4] Regulation of cognition during problem posing — a case study. ***Puay Huat Chua*** (Singapore).
[5] Characterizing the problem-solving processes used by Pupils in classroom: propositioin of a descriptive model. ***Stephane Favier*** (France).
[6] A framework on examining mathematical commuication in problem posing. ***Ling Zhang***, *Jinfa Cai, and Naiqing Song* (China).
[7] Using problem posing to disgnose and understand perservice teachers conceptual understanding. ***Yiling Yao*** *and Jinfa Cai* (China).
[8] Elementary mathematics teachers learning to teach through problem posing:initial findiings of a longitudinal study. ***Dan Zhang***, *Yiling Yao, and Jinfa Cai* (China).
[9] Primary school teachers' behaviors, beliefs, and their interplayt in teaching for problem solving. ***Benjamin Rott*** (Germany).
Session 2
[10] Teaching students how to pose mathematical questions. ***Peter Juhasz***, *Reka Szasz, Lajos Posa, and Ryota Matsuura* (Hungary).
[11] How elementary and middle school teachers formulate multiplication and division word problems. ***Sintria Lautert***, *Alina Galvao Spinillo, Rute Elizabete Borba, Juliana Silva, and Ernani Martins dos Santos* (Brazil).
[12] Gifted students strategy felxibility in non-routine problem solving. ***Yeliz Yazgan*** (Turkey).
[13] Types of reasoning promoted in mathematics classes in the context of problem-solving instruction in Geneva. ***Maud Chanudet*** (France).
[14] Investigating elementary school students' STEM problem posing: the walkstem after-school club. ***Min Wang*** *and Candace Ann Walkington* (USA).
[15] Designing professional development programs that support teachers' incorporation of problem solving in their mathematics instruction —the DCP mode. ***Jillian White***, *Patrick Johnson, and Merrilyn Enid Goos* (Ireland).
[16] Mathematics problem multicontextual exploration, solving and posing in the classroom and teacher education: a perspective in critical education. ***Silvanio de Andrade*** (Brazil).
Session 3
[17] How do Chinese textbooks incorporate mathematical problem posing in different stages? ***Jiajie Yan***, *Yufeng Guo, and Wenjia Zhou* (China).
[18] Appreciation of the aesthetic qualities of mathematical objects: an analysis of students problem sovling. ***Hayato Hanazono*** (Japan).
[19] Towards LITE, a local instructional theory for mathematicall explorations. ***Jayasree Subramanian***, *K. Subramaniam, and R. Ramanujam* (India).
[20] Graphic organizers for problem-solving in primary mathematics: teachers' reflections. ***Nor Azura Abdullah*** (Brunei).
[21] The effect of problem-posing strategies on primary Pre-service teachers conceptual knowledge of fractions. ***Eda Vula*** (Albania).
[22] Investigating mathematics teachers' knowledge for teaching problem-solving. ***Brantina Chirinda*** *and Patrick Barmby* (South Africa).
[23] Elements of mathematical activity that emerge when future teachers of secondary school mathematics use digital technologies to solve problems. ***Matias Camacho-Machin***, *Alexander Hernandez, and Josefa Perdomo-Diaz* (Spain).
[24] A study on evaluating prospective teachers' problem posing activity. ***Zoltan Kovacs*** (Hungary).
[25] Use of video clips to engagestudents in mathematical problem soling. ***Tin Lam Toh*** *and Eng* Guan *Tay* (Singapore).
[26] Problem solving and generalization with an advanced computing environment. ***Marina Marchisio***, *Alice Barana, Alberto Conte, Cecilia Fissore, and Fabio Roman* (Italy).
[27] A study on improving flexibility in problem solving: unit teaching based on big-idea in mathematics. *Hongyun Li, Jian Dun, and* ***Qilei Feng*** (China).

Session 4	
[28]	Supporting students to compress mathematical knowledge while problem solving. ***Rogier Bos*** *and Rona Lemmink* (Netherlands).
[29]	A strategy for enhancing mathematical problem solving. ***Miguel Cruz Ramirez*** (Cuba).
[30]	A study on primary school mathematical problem-posing abilities in China. ***Na Yan*** (China).
[31]	The process of posing problems: development of a descriptive process model for problem posing. ***Lukas Baumanns*** *and Benjamin Rott* (Germany).
[32]	Automation of math discovery support: reinforcement of problems with criteria for evaluating partial solutions. ***Sergei Nickolaevitch Pozdniakov*** (Russia).
[33]	Divison problem posing of fifth graders: a cross-national study in China and the United States. ***Fenqjen Luo****, Yali Yu, Monte Meyerink, and Ciara Burgal* (USA).
[34]	Students engagment in problem posing while solve a fermi problem. ***Nelia Amado****, Susana Carreira, and Monica Alexandra Robelo Valadao* (Portugal).
[35]	Develop your own problem! — problem posing in given real-world situations. ***Luisa-Marie Hartmann****, Stanislaw Schukajlow, and Janina Krawitz* (Germany).

2.1. Teaching and learning in relation to PS/PP

This category contains both empirical and theoretical discussion papers that were classified into several categories

2.1.1. Creating Problems (or PP) in classroom situations

Both Zheng[1] and Harmann et al.[35] discussed the creation of problems by students. Zhang focuses more on the theories and characterization of problems created by middle school students while Harmann et al., to find out the type of problems students can pose and solve, which are problems that are related to the real world. Peter Juhasz et al.[10] propose how best to facilitate problem posing for mainstream students based on a method (which they term Posa method) that was originally developed for gifted students.

2.1.2. Processes involved in PS/PP

Lukas Baumanns and Rott[31] presented a descriptive model of the processes involved in students' problem posing through his empirical studies. Chua[4], through a case study, presented the regulatory cognitive phases during PP of students. Favier[5] characterizes the processes used by students when they solve problems in the mathematics classrooms. Zhang et al.[8] presented a framework for examining mathematical communication in problem posing, which refers to the process of conveying and expressing information during activities of problem posing.

Yazgan[12] examined students' flexibility in solving non-routine problems. Chanudet[13] studied the types of mathematical reasonings that students exhibited in problem solving instruction. Wang[14] presented a paper on investigating students problem posing ability. A study by Yan et al.[17] compared the problem posing ability between Han Chinese student and the students from the minority groups in China. Hanazono[18] discussed how students are able to appreciate the aesthetic qualities of mathematical objects through appropriate teacher intervention discussed in his paper.

Studies on intervention to improve students' processes in PS/PP are presented in this paper. Regarding the flexibility, Li et al.[27] discussed a teaching model that has been shown in their studies to improve students' flexibility in problem solving. Bos and Lemmink[28] presented strategies to compress mathematical objects, procedures and statements, which could play a major role in achieve success in mathematical problem solving. Ramirez[29] presented a strategy, heuristic strategy, for enhancing mathematical problem posing. With the use of a type of modelling problems, the Fermi problems, Amado et al.[34] showed that students' assumptions made of a problem are very closely connected to the types of problems that they pose.

In a study conducted by Luo et al.[33], the similarities and differences of the problems posed on division posed by students from China and the United States were analyzed.

2.1.3. Technology and PS

Three papers on the use of technology to enhance PS/PP were among the paper presentation. Pozdniakov[32] proposed the use of modern computer technologies to support independent PS, based on Polya's model of problem solving as the framework. Marchisio et al.[26] proposed the use of technologies, an ACE (Advanced Computing Environment), to support PS through representing and exploring mathematical tasks. Toh and Tay[25] discussed the creation and adaptation of video clips for the teaching of PS.

2.1.4. Enactment of PS/PP in mathematics classrooms

Subramanian et al.[19] presented a Local Instructional Theory for Exploration for classroom enactment of mathematical exploration. The study was based on the Realistic Mathematics Education framework. Abdullah[20] proposed the use of graphic organizers in developing PS ability amongst students in the Brunei schools.

2.2. Teacher education in relation to PS/PP

Beltran-Joaquin[3] presented a study on pre-service and in-service teachers' PP, she highlighted the need to strengthen PP skills among mathematics teachers. Lautert et al.[11] offered a zoom-in view in studying how elementary and middle school teachers formulate multiplication and division word problems, showing that the teachers could have difficulty posing complex and challenging word problems. Kovacs[24] presented a study on evaluating pre-service teachers' PP activity, by examining how the mathematical background of the original problem changes during PP.

Yao and Cai[7] presented a study and asserted that PP contributes to pre-service teachers' conceptual understanding of division of fractions, but also to diagnose and appreciate their mathematical understanding. Another study by Vula[21] showed that PP resulted in positively impact pre-service teachers' conceptual understanding of fractions. Further, Zhang et al.[8], in their large-scale longitudinal study, claimed PP could be an approach to build teachers' pedagogy in mathematics.

Chirinda and Barmby[22] investigated South African in-service teachers' knowledge for teaching PS using Chapman's MPSKT framework. White et al.[15], through an extensive literature review, summarized into the DCP model into the features of an effective professional development that support infusing of PS into mathematics instruction. The finding of Rott[9] suggests the importance of teachers' beliefs in the success of the teaching of PS. Matias Camacho-Machin et al.[23] discussed the impact on a group of pre-service teachers in using digital technology (GeoGebra) in solving mathematics problems. There was evidence of mathematical creation and reasoning and much activities among the pre-service teachers.

2.3. Textbook analysis in relation to PS/PP

There are two papers on this sub-topic. Wang and Wang[2] presented a historical comparison of the problems and PP tasks of three series of middle school mathematics textbooks in China and found an increasing trend in PP tasks in the textbooks over the years. Yan et al.[17] examines how PP tasks were introduced into the Chinese textbooks in different stages. The nature of the PP tasks was different in the primary and junior high levels.

2.4. Social context in relation to PS/PP

There is one paper on this sub-topic. Andrade[16] in his presentation calls for a PS and PP interconnected approach vis problem exploration in a critical education perspective. PS/PP should be seen, in addition to the pedagogical consideration, from the wider level of socio-political-cultural context, as are the other dimensions of education.

3. What Next?

The TSG-17 had very rich discussion over the four sessions described in the above section. All the papers were compiled into pdf format and made available to all the participants of the TSG. The work of the TSG does not end with the conclusion of the congress. As an afterwork of the congress, selected presenters from the above presentation were invited to contribute their work to (1) a special issue in Hiroshima Journal of Mathematics Education which has been published in October 2022; and (2) a book on PS/PP to be published by Springer which targeted before the next ICME. The authors were invited to refine their papers, which will be peer-reviewed for the publication process.

Acknowledgment

The TSG-17 team would like to thank the organizing committee for this opportunity to helm the TSG on PS/PP. In particular, the chair would like to acknowledge Professor Ed Silver who was the original chair of TSG-17, and started the initial work with the chair in the paper review process prior to the covid-19 pandemic. Professor Silver withdrew from this participation due to health reasons. The TSG-17 team wish he could continue to enjoy a good health.

References

P. Liljedahl and M. Santos-Trigo (Eds.) (2019). *Mathematical Problem Solving: Current Themes, Trends, and Research.* ICME-13 Monographs. Springer.

G. Pólya (1945). *How To Solve It: A New Aspect of Mathematical Method.* Princeton University Press.

M. Santos-Trigo (2019). Mathematical problem solving and the use of digital technologies. In P. Liljedahl, & M. Santos-Trigo (Eds.) *Mathematical Problem Solving: Current Themes, Trends, and Research* (pp. 63–90). Springer.

A. H. Schoenfeld (1985). *Mathematical Problem Solving.* New York: Academic Press.

T. L. Toh, K. S. Quek and E. G. Tay (2008a). *Problem Solving in the Mathematics Classroom (Junior College).* Association of Mathematics Educators.

T. L. Toh, K. S. Quek and E. G. Tay (2008b). Mathematical Problem Solving — A New Paradigm. In J. Vincent, R. Pierce and J. Dowsey (Eds.), *Connected Maths: MAV Yearbook 2008* (pp. 356–365). The Mathematical Association of Victoria.